# Chapter 24

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## Gauss's Law



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# Lecture 02

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## Gauss's Law



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# Gauss's Law, Introduction

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Gauss's law is an expression of the general relationship between the net electric flux through a closed surface and the charge enclosed by the surface.

The closed surface is often called a **Gaussian surface**.



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# Gauss's Law – General

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A positive point charge, $q$, is located at the center of a sphere of radius $r$.

The magnitude of the electric field everywhere on the surface of the sphere is

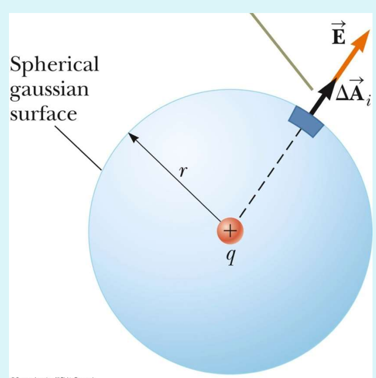
$$E = \frac{1}{4\pi\epsilon_\circ}\frac{q}{r^2}$$

The surface area of a sphere is:

$$A = 4\pi r^2$$

The angle between the directions of ***E*** and ***dA*** is

$$\theta = 0^\circ$$



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# Gauss's Law – General, cont.

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- The field lines are directed radially outward and are perpendicular to the surface at every point.

$$\Phi = \oint \vec{E}_i \cdot d\vec{A}_i = \oint E\, dA = EA$$

- This will be the net flux through the Gaussian surface, the sphere of radius *r*.

$$\Phi = EA = \frac{1}{4\pi\epsilon_\circ}\frac{q}{r^2}\,4\pi r^2$$

$$\Phi_{net} = \frac{q_{in}}{\epsilon_\circ}$$



# Gauss's Law – General

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" The net flux through any closed surface surrounding a net charge, $q_{in}$, is given by

$$\Phi_{net} = \frac{q_{in}}{\epsilon_\circ}$$

and is **independent** of the shape of that surface."

The net electric flux through a closed surface that surrounds no charge is ZERO.

# Gaussian Surface, Example

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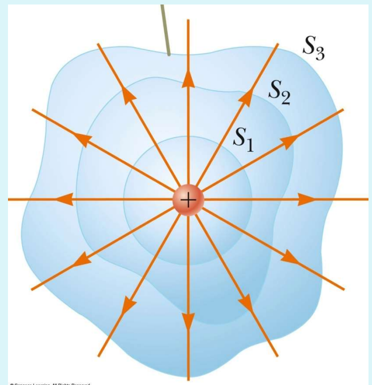
- Closed surfaces of various shapes can surround the charge.
  - Only $S_1$ is spherical
- Verifies the net flux through any closed surface surrounding a point charge q is given by $q/\epsilon_\circ$ and is independent of the shape of the surface.



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# Gaussian Surface, Example 2

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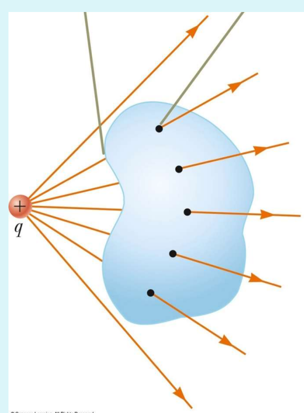
- The charge is outside the closed surface with an arbitrary shape.
- Any field line entering the surface leaves at another point.
- Verifies the electric flux through a closed surface that surrounds no charge is zero.



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# Gauss's Law – Final

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The mathematical form of Gauss's law states

$$\Phi_{net} = \oint \vec{E}_i \cdot d\vec{A}_i = \frac{q_{in}}{\epsilon_\circ}$$

- $q_{in}$ is the net charge inside the surface.
- E represents the electric field at any point on the surface.

# Relating the net enclosed charge and the net flux

Friday, 29 January, 2021 20:49

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.


R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.

J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.

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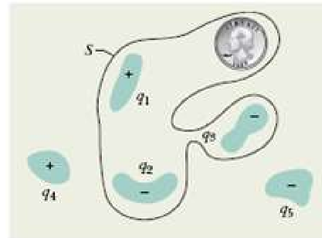
H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.


The figure shows five charged lumps of plastic and an electrically neutral coin. The cross section of a Gaussian surface S is indicated. What is the net electric flux through the surface if $q_1 = q_4 = 3.1\,nC$, $q_2 = q_5 = -5.9\,nC$ and $q_3 = -3.1\,nC$ ?




## Solution

The coin does not contribute to $\Phi$ because it is neutral. Charges $q_4$ and $q_5$ do not contribute because they are outside surface S.

Thus, $q_{enc}$ is only the sum $q_1 + q_2 + q_3$ and:

$$\Phi = \frac{q_{enc}}{\varepsilon_0} = \frac{q_1 + q_2 + q_3}{\varepsilon_0}$$

$$= \frac{+3.1 \times 10^{-9}C - 5.9 \times 10^{-9}C - 3.1 \times 10^{-9}C}{8.85 \times 10^{-12}C^2/N \cdot m^2}$$

$$= -670 N \cdot m^2/C$$

The minus sign shows that the net flux through the surface is **inward** and thus that the net charge within the surface is negative.

# Flux through a closed cube, nonuniform field

Friday, 29 January, 2021 20:50

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
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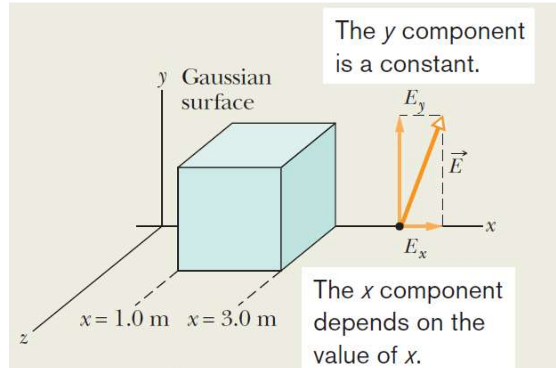
A nonuniform electric field given by $\vec{E} = (3x\hat{\imath} + 4\hat{\jmath})\ N/C$ pierces the Gaussian cube shown in the figure. (x is in meters.)

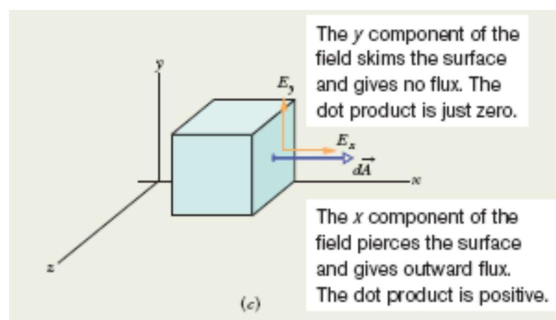
- What is the net electric flux through the surface of the cube?
- What is the net charge enclosed by the Gaussian cube?




## Solution

(A) The right face: An area vector $\vec{A}$ is always perpendicular to its surface and always points away from the interior of a Gaussian surface. Thus, the vector $d\vec{A}$ for any area element on the right face of the cube must point in the positive direction of the $x$ axis.

$$d\vec{A} = dA\hat{\imath}$$




The flux $\Phi_r$ through the right face is then

$$\Phi_r = \int \vec{E} \cdot d\vec{A} = \int (3.0x\hat{\imath} + 4.0\hat{\jmath}) \cdot (dA\hat{\imath})$$

$$= \int [(3.0x)(dA)\hat{\imath} \cdot \hat{\imath} + (4.0)(dA)\hat{\jmath} \cdot \hat{\imath}]$$

$$= \int (3.0xdA + 0) = 3.0 \int xdA$$

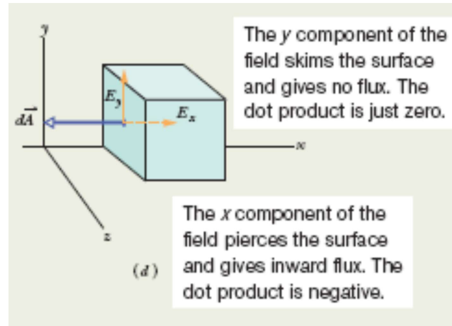
We note that $x$ has the same value everywhere on that face, $x = 3.0m$. This means we can substitute that constant value for $x$. Thus, we have

$$\Phi_r = 3.0 \int (3.0)dA = 9.0 \int dA.$$

The integral $\int dA$ merely gives us the area $A = 4.0m^2$ of the right face; so

$$\Phi_r = (9.0N/C)(4.0m^2) = 36N \cdot m^2/C.$$

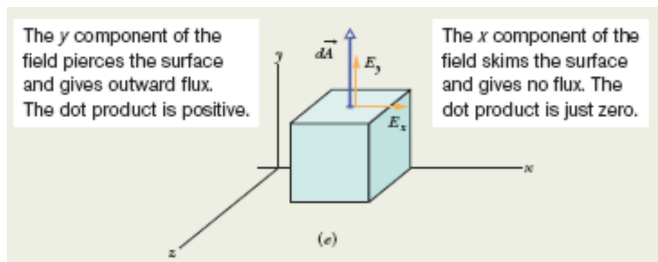
The left face: The differential area vector $d\vec{A}$ points in the negative direction of the $x$ axis, and thus $d\vec{A} = -dA\hat{\imath}$




The term x again appears in our integration, and it is again constant over the face being considered. However, on the left face, $x = 1.0m$

$$\Phi_l = -12N \cdot m^2/C.$$

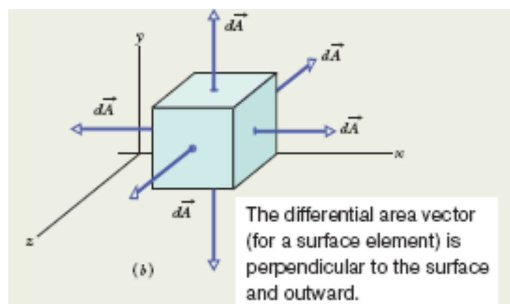
The top face: The differential area vector $d\vec{A}$ points in the positive direction of the y  axis, and thus $d\vec{A} = dA\hat{\jmath}$. The flux $\Phi_t$ through the top face is then:




$$\Phi_t = \int (3.0x\hat{\imath} + 4.0\hat{\jmath}) \cdot (dA\hat{\jmath})$$

$$= \int [(3.0x)(dA)\hat{\imath} \cdot \hat{\jmath} + (4.0)(dA)\hat{\jmath} \cdot \hat{\jmath}]$$

$$= \int (0 + 4.0dA) = 4.0 \int dA = 16N \cdot m^2/C$$




The bottom face, $d\vec{A} = dA\hat{\jmath}$, and we find

$$\Phi_b = -16N \cdot m^2/C.$$

For the **front face** we have $d\vec{A} = dA\hat{k}$, and for the **back face**, $d\vec{A} = -dA\hat{k}$ .

When we take the dot product of the given electric field $\vec{E} = 3.0x\hat{\imath} + 4.0\hat{\jmath}$ with either of these expressions for $d\vec{A}$, we get **ZERO** and thus there is no flux through those faces.

We can now find the total flux through the six sides of the cube:

$$\Phi = (36 - 12 + 16 - 16 + 0 + 0)N \cdot m^2/C = 24N \cdot m^2/C.$$

(B) What is the net charge enclosed by the Gaussian cube?

**Solution**

We use Gauss' law to find the charge $q_{enc}$enclosed by the cube:

$$q_{enc} = \varepsilon_0 \Phi$$

$$= (8.85 \times 10^{-12} C^2/N \cdot m^2)(24N \cdot \frac{m^2}{C})$$

$$= 2.1 \times 10^{-10} C.$$